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ความยาววิ่งเฉลี่ยสำหรับแผนภูมิควบคุมรวมสะสมของกระบวนการ ARMAX(1,1)

Average Run Length for CUSUM control chart of ARMAX(1,1) process

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บทคัดย่อ

วัตถุประสงค์ของการควบคุมกระบวนการทางสถิติ (SPC) คือการตรวจสอบการดำเนินงานในการควบคุมกระบวนการ หนึ่งในเครื่องมือที่มีประสิทธิภาพของ SPC คือ แผนภูมิควบคุมรวมสะสมซึ่งถูกนำไปประยุกต์ใช้อย่างกว้างขวาง เช่น เภสัชกรรม วิศวกรรม เศรษฐกิจ และอื่นๆ หลายกระบวนการที่สนใจค่าสังเกตมักจะมีความสัมพันธ์กัน การวัดผลการดำเนินงานของแผนภูมิควบคุมนั้นจะใช้ค่าความยาววิ่งเฉลี่ย (ARL) เป้าหมายหลักของงานวิจัยนี้เพื่อหาสูตรสำเร็จสำหรับคำนวณค่าความยาววิ่งเฉลี่ยสำหรับแผนภูมิควบคุมรวมสะสมเมื่อค่าสังเกตอยู่ในรูปแบบ ARMAX(1,1) และใช้การแจกแจงแบบเลขชี้กำลัง การตรวจสอบความแม่นยำของผลที่ได้จากสูตรสำเร็จสำหรับคำนวณค่าความยาววิ่งเฉลี่ยกับวิธีปริพันธ์เชิงตัวเลขด้วยกฎของเกาส์พบว่าสูตรสำเร็จสำหรับคำนวณค่าความยาววิ่งเฉลี่ยกับวิธีปริพันธ์เชิงตัวเลขสอดคล้องกันอย่างดีเยี่ยม ข้อเท็จจริงนี้ชี้ให้เห็นว่าสูตรสำเร็จที่ได้มีความแม่นยำสูงอย่างเพียงพอ

ABSTRACT

The objective of Statistical Process Control (SPC) is to monitor the operation of in control process. One of efficient tools of SPC is the Cumulative Sum (CUSUM) control chart which widely used in several of application such as pharmaceutics, engineering, economics and in the other area. For many processes of interest, observations which are closely spaced in time will be correlated. The measure of performance used is the average run length (ARL). The main goal of this paper is to derive explicit formulas for ARL of the CUSUM control chart for ARMAX(1,1) process and using exponential white noise. Checking the accuracy of results, the result obtained from explicit formulas with numerical integral equation by Gauss-Legendre rule were compared. An excellent agreement between the explicit solution and numerical solutions was found. This fact is an additional indication that the explicit formulas are sufficiently high accuracy.

คำสำคัญ: ความยาววิ่งเฉลี่ย แผนภูมิควบคุมรวมสะสม กระบวนการ ARMAX(1,1)

Keyword: Average run length, Cumulative sum control chart, ARMAX(1,1) process.

1. INTRODUCTION

Statistical Process Control (SPC) plays an important role in the quality improvement program of many companies. SPC employs statistical techniques to analyze a process to determine if the process is in a state of "statistical control." Deming (Deming, 1986) defines a process being in a state of control if there is no indication of any special causes of variation. Control charts are one of the major tools of SPC. They play an important role in an overall quality program due to their ability to distinguish between special and common causes of variability. The simplest control chart consists of a center line which is given the value of average quality of the process. Upper and lower control limits are added to the control chart. The control limits are based upon some multiple of the com m on case variability of the quality of the process. These limits are set so that it is very unlikely that a process with only common cause variability will produce a point outside the control limits. A control chart declares a process out of control (or identifies a special cause of variability) if any point lies outside the control limits. A property of control charts that would be very desirable is the ability to quickly detect a special cause. The sooner that special cause is detected, the sooner the quality of that product can be improved. Another very important property would be for the control chart to only declare that a process is out of control when there is a source of special cause variability is present. Due to testing error, sometimes control charts signal that the process is out of control when there is no special causes present. This situation is referred to as a false alarm. Thus, the desirable properties of a control chart would be that it quickly detects special causes when they are present and the control chart would declare few false alarms.

The control chart such as the Shewhart control chart proposed by Shewhart (Shewhart, 1931), the cumulative sum (CUSUM) control chart first proposed by Page (Page, 1954), and the exponentially weighted moving average (EWMA) control chart was initially introduced by Robert (Robert, 1959), these are used to monitor product quality and detect the occurrence of special causes that may by indicated to out of control situations. Both CUSUM and EWMA control charts are based on the assumption that observations being monitored will produce measurements that are independent and identically distribution over time when only the inherent sources of variability are present in the process (Smiley and Keoagile, 2005). There are many situations in which the process is serially correlation such as in chemical processes if the choice of control charts depends on the quality characteristics to be measured in the processes. Hence, these systems have to be monitored by particular control charts.

The Cumulative Sum (CUSUM) control chart is primarily used to maintain (rather than improve) current control of a process (Duncan, 1965). The primary advantage of the CUSUM chart is that it will identify a sudden or persistent change in the process average more rapidly than a Shewhart control chart incorporating the initial Shewhart interpretation rule. Furthermore, it is often possible to pinpoint the exact sample where the change in the process occurred (Wetherill and Brown, 1991). Goldsmith and Whitfield (1991) examined the effectiveness of CUSUM control charts using computer simulation and have derived both OC curves and equations from their studies. The performance of CUSUM control charts in the presence of autocorrelation has been studied in a number of contexts. See, for example, Yashchin (1993), VanBrackle and Reynolds (1997), and Timmer, Pignatiello, and Longnecker (1998). Accordingly, the main goal of paper is to study the Fredholm type integral equations method to derive a closed-form solution of average run length for Autoregressive with explanatory variable (ARMAX(1,1)).

The average run length (ARL) is applied criterion of measures to confirm the performance of a control chart. The frequently used operation characteristics are in control average run length (ARL_0)and out of control average run length (ARL_1). Several methods for evaluating ARL were found out by professionals in the areas of mathematics such as Markov chain approach (MCA), Monte-Carlo simulation (MC) and integral equation approach (IE). In 1959, Robert (Robert, 1959) presented the EWMA control chart by using Monte Carlo simulations technique for evaluating the ARL. Harris and Ross (Harris and Ross, 1991) studied Cumulative Sum with serially correlated observations via Monte Carlo Simulation. Crowder (1987) used integral equations approach for compute ARLs of EWMA charts and found that the integral-equation approach also extends easily to distributions that are nonnormal an important feature that allows use of the approach when studying control procedures for process parameters other than a process mean, the accuracy of this method is very good. Recently, Areepong and Novikov (Areepong and Novikov, 2009) presented that when observation are exponential distribution, the explicit solution of average run length and average delay for EWMA control chart are derived. Mititelu et al. (Mititelu et al, 2010) introduced and explicit solution of ARL by Fredholm integral equation of the second kind for one sided EWMA control scheme. Petcharat et al. (2013) derived explicit formulas of ARL for EWMA and CUSUM control chart when observations are q order Moving Average with exponential white noise by using the Integral Equation. Later Paichit (2016) presented the exact expression of ARL for EWMA control chart for ARX(p) process by used Integral equation.

The main purpose of this paper is to study the analytical and numerical method for the derivation of solution of ARL for CUSUM control chart for ARMAX(1,1) observations with exponential white noise for detecting of a change in process mean. The integral equation technique is used to derive these explicit solution for ARL.

The procedures of the paper are as follows: In section 1, the introduction is presented, Section 2 introduced the CUSUM control chart for ARMAX(1,1) processes. The derivation of explicit formula of ARL is expressed in section 3, the numerical method for solving integral equation to obtain approximation of ARL is presented in section 4, the comparison of results is presented in section 5, the conclusions and discussion of the results is addressed in section 6.

2. THE ARMAX(1,1) PROCESS FOR CUSUM CONTROL CHART

The Cumulative Sum (CUSUM) control chart is primarily used to maintain (rather than improve) current control of a process (Duncan, 1965). The primary advantage of the CUSUM chart is that it will identify a sudden or persistent change in the process average more rapidly than a Shewhart control chart incorporating the initial Shewhart interpretation rule.

Given Y_t be a sequence of the Autoregressive with Explanatory variable: ARMAX (1,1) random processes. The CUSUM processes regress the current value Y_t on the past values of itself Y_{t-1} and past random errors that occurred in past time periods ε_{t-1}. Thus, the current value is a white noise error term.

The definition of CUSUM statistics based on ARMAX (1,1) process is the following recursion:

$$C_t = \max(C_{t-1} + \varepsilon_t - a, 0)\ ; t = 1,2,... \quad (1)$$

where C_t is the CUSUM statistics, ε_t is a sequence of independent and identically distribution random variables. The value of C_0 is an initial value of CUSUM statistics, C_0 = *u* and *a* is non-zero constant.

The Mixed Autoregressive–Moving Average Processes with Explanatory variable: ARMAX (1,1) processes can be written as:

$$Y_t = \phi_1 Y_{t-1} + \left(X_t - \beta_1 X_{t-1}\right) + \varepsilon_t - \theta_1 \varepsilon_{t-1} \quad (2)$$

where ε_t is to be a white noise processes assumed with exponential distribution. The initial value is normally to be the process mean, an autoregressive coefficient $-1 < \phi_1 < 1$ and a moving average coefficient $-1 < \theta_1 < 1$. It is assumed that the initial value of ARMAX (1,1) processes $Y_{t-1} = 1$ and $X_t, X_{t-1} = 1$ are explanatory variables.

In this paper, the case of positive change in distribution which crossing the upper control limit raises alarm is mainly discussed. Given $\varepsilon_t, t = 1,2,...$ is a sequence of independent identically distribution random variables with exponential parameter (α). It is normally assumed that under in control state, the parameter has known in-control value $(\alpha = \alpha_0)$. The parameter α could be changed to out-of-control value $(\alpha = \alpha_1)$, when $(\theta = \infty)$, is the change-point time.

The first passage times for the CUSUM can be written as:

$$\tau_h = \inf(t > 0 : C_t > h),\ h > u \quad (3)$$

Where τ_h is a stopping time

H is a constant parameter known as upper control Limit (UCL).

The most two characteristics control chart are ARL_0 and ARL_1 as following:

$$ARL_0 = E_\infty(\tau_h) \quad (4)$$

$$ARL_1 = E_\theta(\tau_h - \theta + 1 \mid \tau_h \geq \theta) \quad (5)$$

where

$E_\infty(.)$ is the expectation corresponding to the target value and is assumed to be large enough.

$E_\theta(.)$ is the expectation under the assumption that change-point occurs at time $\theta = 1$.

3. EXPLICIT SOLUTION OF CUSUM CONTROL CHART FOR ARMAX(1,1) PROCESSES

The notations $\mathbf{P_c}$ denote the probability measure and $\mathbf{E_c}$ denote the expression corresponding chart after it is reset at $u \in [0,h]$. Let $H(u) = \mathbf{E}(\tau_h)$ be the ARL of CUSUM control chart after it is reset at $u \in [0,h]$. The solution of integral equation is as following

$H(u)$ = 1+$\mathbf{E_c}$[I{0 < C_1 < b}$H(C_1)$]+$\mathbf{P_c}${C_1=0}$H(0)$. (6)

Therefore, the integral equation of CUSUM control chart for ARMAX(1,1) process is

$$Y_t = \phi_1 Y_{t-1} + \left(X_t - \beta_1 X_{t-1}\right) + \varepsilon_t - \theta_1 \varepsilon_{t-1}.$$

So,

$$H(u) = 1 + \alpha e^{\alpha\left(u-a+\phi_1 Y_{t-1}+\left(X_t-\beta_1 X_{t-1}\right)-\theta_1\varepsilon_{t-1}\right)} \int_0^h H(w) e^{-\alpha w} dw + \left(1 - e^{-\alpha\left(a-u-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)}\right) H(0). \quad (7)$$

Let $k = \int_0^h H(w) e^{-\alpha w} dw.$ Consequently, $H(u)$ can be rewritten as

$$H(u) = 1 + \alpha e^{\alpha\left(u-a+\phi_1 Y_{t-1}+\left(X_t-\beta_1 X_{t-1}\right)-\theta_1\varepsilon_{t-1}\right)} k + \left(1 - e^{-\alpha\left(a-u-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)}\right) H(0). \quad (8)$$

In particular at u=0, we obtain $H(0)$ as following form

$$H(0) = 1 + \alpha e^{\alpha\left(-a+\phi_1 Y_{t-1}+\left(X_t-\beta_1 X_{t-1}\right)-\theta_1\varepsilon_{t-1}\right)} k + \left(1 - e^{-\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)}\right) H(0).$$

$$= e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} + \alpha k.$$

Substituting $H(0)$ into Equation (8) , then $H(u)$ as following form

$$H(u) = 1 + \alpha e^{\alpha\left(u-a+\phi_1 Y_{t-1}+\left(X_t-\beta_1 X_{t-1}\right)-\theta_1\varepsilon_{t-1}\right)} k + \left(1 - e^{-\alpha\left(a-u-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)}\right) \times e^{-\alpha\left(a-u-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} + \alpha k.$$

Consequently,

$$H(u) = 1 + \alpha k + e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} - e^{\alpha u}$$

To find a constant k as following form

$$k = \int_0^h H(w) e^{-\alpha w} dw$$

$$= \frac{e^{\alpha h}}{\alpha}\left(1 - e^{-\alpha h}\right)\left(1 + e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)}\right) - h e^{\alpha h}.$$

Substituting a constant k into Equation (8) as follows

$$H(u) = 1 + \alpha\left(\frac{e^{\alpha h}}{\alpha}\left(1 - e^{-\alpha h}\right)\left(1 + e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)}\right) - h e^{\alpha h}\right) + e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} - e^{\alpha u}$$

$$= e^{\alpha h}\left(1 + e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} - \alpha h\right) - e^{\alpha u}.$$

Thus, we get the explicit solution for ARL of CUSUM control chart as follow

$$H(u) = e^{\alpha h}\left(1 + e^{\alpha\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} - \alpha h\right) - e^{\alpha u}.$$

Since the process is in-control state with exponential parameter $\alpha=\alpha_0$, we obtain the explicit solution for ARL_0 as follows

$$ARL_0 = e^{\alpha_0 h}\left(1 + e^{\alpha_0\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} - \alpha_0 h\right) - e^{\alpha_0 u}.$$

Since the process is out-of-control state with exponential parameter $\alpha=\alpha_1$, The explicit solution for ARL_1 can be written as follows

$$ARL_1 = e^{\alpha_1 h}\left(1 + e^{\alpha_1\left(a-\phi_1 Y_{t-1}-\left(X_t-\beta_1 X_{t-1}\right)+\theta_1\varepsilon_{t-1}\right)} - \alpha_1 h\right) - e^{\alpha_1 u}.$$

where $-1<\phi_1<1$ is Autoregressive coefficient, $-1<\theta_1<1$ is a moving average coefficient, $X_{1,} X_{t-1}$ is Explanatory variable, *h* is a upper control limit and ε_{t-1} is initial values of ARMAX(1,1) process.

4. NUMERICAL INTEGRAL EQUATION

Generally, the Integral Equation could not be analytically solved *H*(*u*) and it is necessary to use numerical methods to solve them. Kantorovich and krylov (Kantorovich and krylov, 1958); Atkinson and Han (Atkinson and Han, 2001) have been developed numerical schemes for solving integral equation. We shall use a quadrature rule to approximate the integral by finite sum of area of rectangles with based on *h/m* beginning at zero. Particularly, once the choice of a quadrature rule is made, the interval [0,*h*] is divided into a partition $0 \le a_1 \le a_2 \le \ldots \le a_m \le m$ and set of constant weighted $w_j = (h/m) \ge 0.$

The approximation for an integral is of the form:

$$\int_0^b W(x) f(x) dx \approx \sum_{j=1}^{m} w_j f(a_j)$$

where $a_j = \frac{h}{m}\left(\frac{2j-1}{2}\right)$ and $w_j = \frac{h}{m}; j = 1,2,\ldots,m$.

Let $\tilde{H}(u)$ denote to the numerical approximation to integral equation $H(u)$, which can be found as the solution of linear equation as follows:

$$\tilde{H}(a_i) = 1 + \tilde{H}(a_1)F\left(a - a_i - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1}\right) + \theta_1 \varepsilon_{t-1}\right)$$
$$+ \sum_{j=1}^{m} w_j \tilde{H}(a_j) f(a_j + a - a_i - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1}\right) + \theta_1 \varepsilon_{t-1}).$$

The above equation is a system of *m* linear equations in the *m* unknowns $\tilde{H}(a_1), \tilde{H}(a_2), \ldots, \tilde{H}(a_m),$ which can be rearranged as

$$\tilde{H}(a_1) = 1 + \tilde{H}(a_1)[F\left(a - a_1 - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1}\right) + \theta_1 \varepsilon_{t-1}\right)$$
$$+ w_1 f\left(a - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1}\right) + \theta_1 \varepsilon_{t-1}\right)]$$
$$+ \sum_{j=2}^{m} w_j \tilde{H}(a_j) f(a_j + a - a_1 - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1}\right) + \theta_1 \varepsilon_{t-1})$$
$$\vdots$$
$$\tilde{H}(a_m) = 1 + \tilde{H}(a_1)[F((a - a_m - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1}\right) + \theta_1 \varepsilon_{t-1}$$

$$+ \; w_1 f(a_1 + a - a_m - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1} \right) + \theta_1 \varepsilon_{t-1})]$$

$$+ \sum_{j=2}^{m} w_j \tilde{H}(a_j) f(a_j + a - a_m - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1} \right) + \theta_1 \varepsilon_{t-1}).$$

Solving set of equations for the approximate values $\tilde{H}(a_1), \tilde{H}(a_2), \ldots \tilde{H}(a_n)$, the numerical integration for function *H*(*u*) is

$$\tilde{H}(u) = 1 + \tilde{H}(a_1) F(a - u - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1} \right) + \theta_1 \varepsilon_{t-1})$$

$$+ \sum_{j=1}^{m} w_j \tilde{H}(a_j) f((a_j + a - u - \phi_1 Y_{t-1} - \left(X_t - \beta_1 X_{t-1} \right) + \theta_1 \varepsilon_{t-1},$$

with $a_j = \frac{h}{m}\left(\frac{2j-1}{2}\right)$ and $w_j = \frac{h}{m}; j = 1,2,\ldots,m$.

5. COMPARISON RESULTS OF CUSUM CONTROL CHARTS BY EXACT EXPRESSION AND NUMERICAL INTEGRAL EQUATION METHODS

In this section, the results of ARL_0 and ARL_1 for ARMAX (1,1) processes, which are obtained from the exact expression with numerical solution of integral equation method are compared. The results of ARL are expressed in Table 1 to Table 3. The parameter value for in-control parameter $\alpha_0 = 1$ and parameter for out-of-control $\alpha_1 = 1.01$, 1.02, 1.03, 1.04, 1.05, 1.06, 1.07, 1.08, 1.09, 1.10, 1.3, 1.5, 3, and 5 respectively. The performance of the purposed exact expression is considered by the computational times and the absolute percentage difference.

$$Diff(\%) = \frac{\left|\tilde{H}(u) - H(u)\right|}{H(u)} \times 100\%.$$

The results from Table 1 and Table 2 present that these methods are in good agreement. The analytical results agree with numerical approximation with an absolute percentage difference less than 0.05% for m = 1,500 iterations and for computational times of approximately 50 second. The computational times for the proposed analytical explicit solution are less than 1 second.

Table 1. Comparison of ARL_0 and ARL_1 of CUSUM control chart by explicit solution with numerical integral equation for ARMAX(1,1) process with $\phi_1 = 0.1, \theta_1 = 0.1$ .and $\beta_1 = 0.1$

Parameter values of CUSUM chart a = 3, u =1 and h = 4.35			
α	Exact expression	Numerical IE (Time used)	Diff (%)
1.00	370.431	370.43 (56.18)	0.00027
1.01	345.454	345.453 (57.28)	0.00029
1.02	322.608	322.6071 (57.53)	0.00028
1.03	301.68	301.679 (56.07)	0.00033
1.04	282.482	282.4814 (55.43)	0.00021
1.05	264.844	264.8434 (55.34)	0.00023
1.06	248.619	248.6184 (56.18)	0.00024
1.07	233.672	233.6714 (57.28)	0.00026
1.08	219.885	219.8845 (57.35)	0.00023
1.09	207.151	207.1504 (55.47)	0.00029
1.10	195.375	195.3744 (56.38)	0.00031
1.30	74.0401	74.0399 (55.22)	0.00025
1.50	37.2212	37.2211 (56.18)	0.00027
3.00	5.27084	5.27082 (57.13)	0.00025
5.00	2.72168	2.72167 (56.48)	0.00029

Table 2. Comparison of ARL_0 and ARL_1 of CUSUM control chart by explicit solution with numerical integral equation for ARMAX(1,1) process with $\phi_1 = 0.1, \theta_1 = 0.1$ .and $\beta_1 = 0.2$.

Parameter values of EWMA chart a = 3, u=1 and h = 4.151			
α	explicit solution	Numerical IE (Time used)	Diff (%)
1.00	370.267	370.2661 (57.18)	0.00024
1.01	345.929	345.928 (56.25)	0.00029
1.02	323.625	323.624 (56.42)	0.00031
1.03	303.154	303.153 (57.03)	0.00033
1.04	284.338	284.3371 (56.54)	0.00032
1.05	267.021	267.0203 (55.28)	0.00026
1.06	251.061	251.0603 (56.32)	0.00028
1.07	236.332	236.3313 (55.51)	0.00030
1.08	222.722	222.7214 (56.15)	0.00027
1.09	210.129	210.1284 (55.47)	0.00029
1.10	198.465	198.4644 (56.16)	0.00030
1.30	76.7527	76.7524 (56.31)	0.00039
1.50	38.9158	38.9157 (57.14)	0.00026
3.00	5.37993	5.37991 (56.31)	0.00028
5.00	2.72965	2.72964 (56.44)	0.00031

6. CONCLUSION

Explicit solution for ARL of CUSUM control chart in the case of ARMAX(1,1) process with exponential white noise are derived, These formulas are very accurate, and easy to calculate and program. More specifically, the explicit solution take computational time much less than the numerical integral equation.

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